Palindrome algorithm notation by Daniel Marschall

This script shows my notation of the palindrome algorithm, which can transform a number into a palindrome, except for the Lychrel numbers.

a is the number where we want to create a palindrome from:

$$a \in \mathbb{Z}$$

Now we use a recursive function to describe the palindrome-algorithm:

$$f_a(0) \coloneqq a$$

$$f_a(x) = f_a(x-1) + rev[f_a(x-1)]$$

$$x \in \mathbb{N} \setminus \{0\}$$

$rev[]$ is a function that inverses a number. I found no mathematical notation for this function.

For the smallest x (that means the first palindrome that was found) that has following conditions

$$f_a(x) = rev[f_a(x)]$$

$$x \in \mathbb{N} \setminus \{0\}$$

we can define:

$$I(a) \coloneqq x$$

$$P(a) \coloneqq f_a(x)$$

$P(a)$ is the palindrome that results from the intial number a after $I(a)$ iterations with the palindrome-algorithm.

Examples:

$$I(5273) = 1$$
$$P(5273) = 8998$$

$$I(4753) = 5$$
$$P(4753) = 475574$$

There are numbers, which seem to have no solution with the palindrome algorithm. They are called Lychrel numbers. Let's give them a mathematical set notation.

$$\mathbb{L} = \{196, 295, 394, 493, 592, 689, 691, 788, \dots\}$$

$$\mathbb{L} \in \mathbb{N} \setminus \{0\}$$

And maybe we can give all palindrome numbers also a set notification:

$$\mathbb{P} = \{11, 22, \dots, 121, 131, \dots\}$$

$$\mathbb{P} \in \mathbb{N}$$

We can also define:

$$I(x) \in \mathbb{N} \setminus \{0\}$$

$$P(x) \in \mathbb{P}$$

A problem of the palindrome algorithm, the palindrome set and the Lychrel set is, that the palindrome-character is dependent on the base b (decimal has $b = 10$) of the respective number.

So, I suggest that the Lychrel set notification should be extended with the respective base b:

$$\mathbb{L}_{10} = \{196, 295, 394, 493, 592, 689, 691, 788, \dots\}$$

$$\mathbb{L}_b \in \mathbb{N} \setminus \{0\}$$

Also the palindrome set notification should be extended with the base b:

$$\mathbb{P}_{10} = \{11, 22, \dots, 121, 131, \dots\}$$

$$\mathbb{P}_b \in \mathbb{N}$$

Because the inverse function $rev[]$ is also depended on the base b, we can notify it as $rev_b[]$.

The recursive algorithm could be written as:

$$f_a^b(0) \coloneqq a$$

$$f_a^b(x) = f_a^b(x-1) + rev_b[f_a^b(x-1)]$$

For the smallest x that makes the condition

$$f_a^b(x) = rev_b[f_a^b(x)]$$

true, we can define:

$$I_b(a) \coloneqq x$$

$$P_b(a) \coloneqq f_a^b(x)$$

If we believe that there is no solution for the Lychrel numbers, we could define:

$$a \notin \mathbb{L}$$

But there is no mathematical argument that certifies that there is no solution for the Lychrel numbers.

Here is one example for a solution with $b = 16$:

$$I_6(AB) = 3$$

$$P_6(AB) = 6C6$$

Please send me a note at info@daniel-marschall.de if you have improvement suggestions.

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